MOISTURE TRANSFER CHARACTERISTICS IN ANNULAR VERTICAL CHANNELS

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UDC 532.54

Moisture transfer in an annular vertical channel is considered. Analytic relations are obtained for velocity and concentration fields, as well as for moisture-transfer rate.

To dry out the cavities of some engineering devices by means of hard sorbents the estimates are needed of the characteristics of moisture transfer (such as velocity and concentration distributions, moisture-transfer rate, etc.). In the present article results are given of solving this problem for a cavity formed by concentric cylinders of finite length (Fig. 1). On the section z = 0 the source of moisture input is positioned, and on the section z = h a moisture dryer is placed.

If it is assumed that the flow in the channel is laminar and steady-state and that the concentration gradient is constant with respect to height ($\partial C/\partial z = A$), then the processes of free convection are described by a system of homogeneous linear equations [1]:

$$\nu\Delta w - g\beta_1 C = 0, \tag{1}$$

$$Aw = D\Delta C \tag{2}$$

with the following boundary conditions:

1) inside the annular channel the velocities and concentrations stay finite, continuous, and single-valued together with the necessary number of their derivatives;

2) at the channel walls there exist layers of fluid adhering to them with vanishing velocities within them:

$$w|_{\rho=R_1} = 0, \; w|_{\rho=R_2} = 0;$$

2') inside an adhering layer the concentrations suffer no jump;

3) outside the channel there exist neither sources nor sinks of moisture;

4) $C|_{\substack{\rho=R_1\\ z=0}} = C_{max}$.

A cross section of the channel can be defined in the polar (ρ, ψ) coordinate system as $R_2 \le \rho \le R_1$, $0 \le \psi \le 2\pi$. By denoting

$$r = \rho - R_2, \; \varphi = \psi, \; R = R_1 - R_2,$$

one obtains

$$0 \leqslant r \leqslant R, \; 0 \leqslant \varphi \leqslant 2\pi,$$

$$w|_{r=R} = w|_{r=0} = 0, \tag{3}$$

$$C|_{\substack{r=R\\ z=0}} = C_{max}. \tag{3'}$$

Eliminating C from (1) and (2), one finds

$$\Delta\Delta w - k^4 w = 0, \tag{4}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, No. 1, pp. 24-28, January, 1975. Original article submitted March 22, 1974.

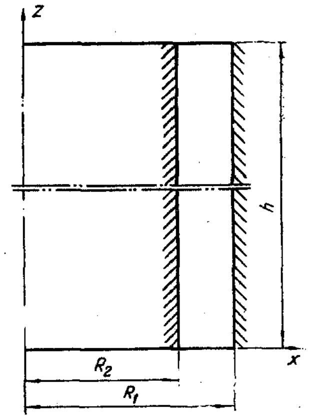

Fig. 1. Computation scheme.

where

$$k^4 = \frac{g\beta_1 A}{\nu D}.$$

The relation (4) is valid only if one of the equalities

$$\Delta w_1 = k^2 w_1;\ \Delta w_2 = -k^2 w_2.$$

is satisfied.

As a solution of (4) one can adopt, for example,

$$w = w_1 + w_2 = w(x,\ y).$$

In polar coordinates

$$\Delta w_1 = \frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial w_1}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 w_1}{\partial \varphi^2} = k^2 w_1.$$

Setting $w_1 = V(r)\Phi(\varphi)$ and using the Fourier method, one obtains

$$-\frac{r^2 V'' + rV' - k^2 r^2 V}{V} = \frac{\Phi''}{\Phi} = -\lambda^2.$$

From the physical considerations (and, in particular, from the boundary conditions $w|_{r=0} = 0$), as well as from symmetry conditions, one has

$$r^2 V'' + rV' - (k^2 r^2 + 1)V = 0. \tag{5}$$

If one denotes $ikr = p$, then Eq. (5) assumes the form

$$p^2 \frac{\partial^2 V}{\partial p^2} + p\frac{\partial V}{\partial p} + (p^2 - 1)V = 0. \tag{6}$$

Equation (6) is a Bessel equation of the first order. Its solution is

$$V = d_1 J_1(p) + d_2 N_1(p),$$

where $J_1(p)$ is the Bessel function of the first kind of order 1; $N_1(p)$ is the Neumann function.

Thus,

$$w_1 = c_1 J_1(ikr) + c_2 N_1(ikr).$$

Similarly,

$$w_2 = c_3 J_1(kr) + c_4 N_1(kr).$$

Finally, using the boundary conditions 1, one has

$$w = c_1 J_1(ikr) + c_3 J_1(kr). \tag{7}$$

It is more convenient to obtain the expression for velocity in the form

$$w = B\left[\frac{J_1(ikr)}{J_1(ikR)} - \frac{J_1(kr)}{J_1(kR)}\right], \tag{8}$$

where the boundary conditions 2 are automatically satisfied.

In accordance with the assumption made above, one has $C = Az + C(x,\ y)$.

It follows from (1) that

$$C(x,\ y) = \frac{\nu}{g\beta_1}\Delta w = \frac{\nu}{g\beta_1}(\Delta w_1 + \Delta w_2),$$

and hence

$$C = Az + \frac{\nu k^2}{g\beta_1}(w_1 - w_2).$$

Employing (8),

$$C = Az + \frac{\nu k^2 B}{g\beta_1}\left[\frac{J_1(ikr)}{J_1(ikR)} + \frac{J_1(kr)}{J_1(kR)}\right].$$

TABLE 1. Geometrical Characteristics and Test Conditions

Parameter	Model			
	A	Б	В	Г
Hydraulic diameter, m	0,01	0,0034	0,055	0,0149
Overall height of partitions, m	0,0025	0,02	0,065	0,5
Distance between partitions, m	0,85	0,58	8,5	5,8
Gap width, m	0,045	0,0225	0,125	0,05
Concentration difference kg/m³	0,00741	0,013	0,00224	0,00159
$kR = (GrPr)^{0.25}$	3,16	1,71	8,4	7,2

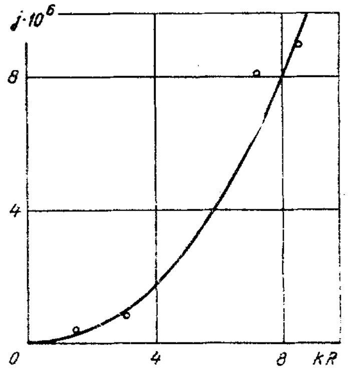

Fig. 2. Moisture-transfer rate vs the criterion kR = $(GrPr)^{0.25}\cdot j$, kg/sec.

From the condition (3')

$$C_{max} = 2\frac{\nu k^2 B}{g\beta_1},$$

and, hence,

$$B = C_{max}\frac{g\beta_1}{2\nu k^2}. \qquad (9)$$

The rate of moisture transfer for the section z = 0 is

$$j = \iint_{(s)} w\varkappa C_B C dx dy = \frac{\varkappa C_B g\beta_1}{4\nu k^2} C_{max}^2 \iint_{(s_1)} \left\{\left[\frac{J_1(ikr)}{J_1(ikR)}\right]^2 - \left[\frac{J_1(kr)}{J_1(kR)}\right]^2\right\} r dr d\varphi. \qquad (10)$$

To evaluate the double integral in (10) one uses familiar recurrence relations together with the equality [2]:

$$\int_0^l z J_\nu^2\left(\frac{m}{l}z\right) dz = \frac{l^2}{2}\left[J_\nu'^2(m) + \left(1 - \frac{\nu^2}{m^2}\right) J_\nu^2(m)\right].$$

Finally, one has

$$j = -\frac{\varkappa C_B \pi D}{4A} C_{max}^2 \left\{\left[\frac{kRI_0(kR)}{I_1(kR)}\right]^2 + \left[\frac{kRJ_0(kR)}{J_1(kR)}\right]^2 - 2\left[\frac{kRI_0(kR)}{I_1(kR)} + \frac{kRJ_0(kR)}{J_1(kR)}\right]\right\},$$

where $I_\nu(m) = i^{-\nu} J_\nu(im)$ is the modified Bessel function of the first kind of order ν.

The obtained formulas were employed to determine the rate of moisture transfer in constructions which can be represented by annular vertical channels with partitions of complex form. The characteristics of the studied objects are shown in the Table 1.

It was established that the effect on the rate of moisture transfer was not so much noticeable as regards the height of the models as the characteristics of the gaps in the partitions. By reducing the length of the model six times (and by a proportional reduction of the distance between the partitions) the rate of moisture transfer doubled, whereas in the experiments carried out on the same model but without partitions the rate increased about 10 times. This shows that in the computations of this and similar cases one should use for the characteristic dimension the hydraulic diameter of the gap in the partitions, and for h the overall height of the partitions should be used. The comparision of these computations and experimental data shows a satisfactory agreement (Fig. 2).

NOTATION

x, y, z	are the rectangular coordinates;
ρ, ψ	are the polar coordinates;
h	is the height;
R_1, R_2	are the external and internal radii, respectively, of the channel;

C is the concentration;
w is the velocity;
g is the acceleration of gravity;
$\varkappa$ is the density;
ν is the kinematic viscosity;
β_1 is the coefficient of volume expansion due to change of concentration;
D is the diffusion coefficient;
C_B is the moisture capacity.

LITERATURE CITED

1. G. A. Ostroumov, Free Convection under Conditions of Internal Problem [in Russian], GTTL, Moscow—Leningrad (1952).
2. D. S. Kuznetsov, Special Functions [in Russian], Vysshaya Shkola, Moscow (1965).